PROPAGATION RATE AND LIMITS OF EXISTENCE OF A TURBULENT FLAME

V. S. Baushev and V. N. Vilyunov

UDC 536.46:533.6

In a theoretical study of turbulent burning it is usually assumed that the average rate of the chemical reaction (heat release) is determined only by the average temperature. Ya. B. Zel'dovich [1] and later T. Karman [2] noted the necessity of taking into account the effect of temperature pulsations on the reaction rate. A quantitative estimate of this effect on the reaction rate constant is given in [3]. A critical analysis of various approaches to the theoretical study of turbulent flames is given in the reviews [4, 5]. In the present article, it is shown that, taking the pulsation component of the temperature and concentration into account, the average rate of the chemical reaction depends on the gradient of the mean temperature and the scale of the turbulent pulsations. The case, where a first-order reaction takes place in the flame is studied in detail. Existence and uniqueness theorems which determine the limits of the propagation of flames are proven. Quantitative rules for the propagation rate, limit, and structure of a turbulent flame front are analyzed with respect to the results of a numerical calculation of a series of variants. Dimensional interpolation equations are presented for the total propagation rate of a flame.

1. Statement of the Problem

In a statistical averaging approach to the description of a one-dimensional diffusion-thermal model of the propagation of a turbulent flame in the presence of a similarity in temperature and concentration (equality of the total coefficients of heat and mass transfer of laminar and turbulent flames, $a_0 + a_1 = D_0 + D_1$) and ignoring thermal expansion, we proceed from the equation

$$\left[\frac{d^2}{d\xi^2} - \omega_1 \frac{d}{d\xi}\right] \langle\theta\rangle = \theta_0^{1-n} \langle\Phi(\theta)\rangle \tag{1.1}$$

with the conditions

$$\xi = -\infty,\ \langle\theta\rangle = \theta_0;\ \xi = +\infty,\ \langle\theta\rangle = 0$$

where $\Phi(\theta)$ is the real rate of the chemical reaction

$$\Phi(\theta) = \theta^n \exp\left(\frac{-\theta}{1-\beta\theta}\right) \tag{1.2}$$

$\langle\theta\rangle$, $\langle\Phi(\theta)\rangle$, ξ, and ω_1 are the dimensionless average temperature, average chemical reaction rate, coordinate, and kinetic propagation rate of a turbulent flame, respectively. The connections between the dimensionless and dimensional values, the scale of measurement, and the parameters of the problem are determined by the equalities

$$\langle\theta\rangle = \frac{E(T_+ - \langle T\rangle)}{RT_+^2},\quad \xi = \frac{x}{x_+},\quad \beta = \frac{RT_+}{E}$$

Tomsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 65-76, May-June, 1972. Original article submitted October 21, 1971.

$$x_+ = [(a_0 + a_1)\tau_+]^{1/2}, \quad \tau_+ = \rho^{1-n} z_0^{-1} \exp\left(\frac{E}{RT_+}\right)$$
$$\theta_0 = \frac{E(T_+ - T_-)}{RT_+^2}, \quad \omega_1 = w_1 (a_0 + a_1)^{-1/2} \tau_+^{1/2} \tag{1.3}$$

Here, T is the temperature, x is the Eulerian coordinate, a and D are the thermal conduction and diffusion coefficients, respectively, ρ is the density, z_0 is the preexponential factor, E is the activation energy, R is the gas constant, n is the order of the reaction, w is the propagation rate of the flame, and τ_+ is the characteristic time of the reaction. Parameters characterizing laminar and turbulent burning are denoted by the indices 0 and 1, respectively, while plus and minus are indices denoting values which characterize the reaction product and the original mixture.

The left-hand part of the differential operator in Eq. (1.1) is obtained on the basis of Reynolds averaging. Averaging the right-hand part is particularly difficult because the function of the instantaneous chemical reaction rate (1.2) is essentially nonlinear. Several approaches are possible here. The most general approach consists in the expansion of $\langle \Phi(\langle\theta\rangle + \theta')\rangle$ in a Taylor series and the subsequent application of Reynolds rules

$$\langle \Phi(\langle\theta\rangle + \theta')\rangle = \langle \Phi(\langle\theta\rangle) + \dot{\Phi}(\langle\theta\rangle)\theta' + $$
$$+ {}^1/_2 \ddot{\Phi}(\langle\theta\rangle)\theta'^2 + \ldots\rangle = \Phi(\langle\theta\rangle) + {}^1/_2 \ddot{\Phi}(\langle\theta\rangle)\langle\theta'^2\rangle + \ldots \tag{1.4}$$

Here, and afterward dots denote differentiation with respect to θ, and a dash denotes turbulent pulsation.

With some assumptions relative to the chief moments of the pulsations, summation series are obtained which considerably facilitate a theoretical analysis of turbulent burning. The chief terms of the expansion (1.4) are presented in [6]. Another approximating approach is the use of the simplest rule of mean arithmetic averaging

$$2\langle\Phi(\theta)\rangle = \Phi(\langle\theta\rangle + \sqrt{\overline{\theta'^2}}) + \Phi(\langle\theta\rangle - \sqrt{\overline{\theta'^2}}) \tag{1.5}$$

The averaging (1.5) satisfies all the Reynolds rules. The averaging (1.5) was used in [3] for a zero-order reaction. In the general case of Eq. (1.2), we have

$$2\langle\Phi(\theta)\rangle = (\langle\theta\rangle + \sqrt{\overline{\theta'^2}})^n \exp\left(-\frac{\langle\theta\rangle + \sqrt{\overline{\theta'^2}}}{1 - \beta(\langle\theta\rangle + \sqrt{\overline{\theta'^2}})}\right) + (\langle\theta\rangle - \sqrt{\overline{\theta'^2}})^n \exp\left(-\frac{\langle\theta\rangle - \sqrt{\overline{\theta'^2}}}{1 - \beta(\langle\theta\rangle - \sqrt{\overline{\theta'^2}})}\right) \tag{1.6}$$

In particular, if $\langle\theta'^m\rangle = 0$ for even m and $\langle\theta'^m\rangle = (\sqrt{\theta'^2})^m$ for odd m, then, for $\beta = 0$ averaging by means of (1.6) coincides with averaging by the series expansion method (1.4).

The mean square pulsation component of the temperature in Eq. (1.6) according to the theory of the mixing process is expressed through gradients of the averaged temperature

$$\sqrt{\overline{\theta'^2}} = \frac{l_1}{x_+}\left|\frac{d\langle\theta\rangle}{d\xi}\right|, \quad \sqrt{\overline{\theta'^2}} = \frac{E\sqrt{\overline{T'^2}}}{RT_+^2} \tag{1.7}$$

where l_1 is the thermal scale of the pulsations. Thus, the mean chemical reaction rate, as opposed to the actual rate, depends not only on the temperature, but on the gradient and relative scale of the turbulent pulsations $F = l_1/x_+$.

The parameter F characterizes the kinetic and hydrodynamic properties of turbulent burning. Assuming that $a_1 = l_1\sqrt{w'^2}$, where $\sqrt{w'^2}$ is the mean square pulsation component of the flux rate, we obtain

$$\frac{1}{F^2\omega_0^2} = \frac{l_0}{l_1}\left[\frac{l_0}{l_1} + \frac{\sqrt{\overline{w'^2}}}{w_0}\right] \tag{1.8}$$

$$l_0 = a_0 / w_0, \quad \omega_0^2 = l_* / l_0, \quad l_* = w_0\tau_+$$

Here, ω_0 is the dimensionless laminar burning rate, l_0 is the thermal width of the laminar flame front, and l_* is the chemical width of the flame front. For large-scale turbulence with $\sqrt{w'^2}/w_0 \gg l_0/l_1$, F is related to the well-known parameters of turbulent burning of K. I. Shchelkin [7] and Kovazhnii [8]

$$\tau_1 / \tau_+ = l_1 w_0 / l_* \sqrt{\overline{w'^2}}, \quad \tau_1 / \tau_0 = l_1 w_0 / l_0 \sqrt{\overline{w'^2}}$$

and in particular

$$\tau_1 / \tau_+ = F^2, \; \tau_1 / \tau_0 = \omega_0{}^2 F^2$$

Here, $\tau_1 = l_1/\sqrt{w'^2}$ is the turbulent mixing time, and $\tau_0 = l_0/w_0$ is the thermal relaxation time of the laminar flame.

We shall examine the first-order reaction in detail later. Shifting to a new variable (for brevity, we shall drop the averaging sign and the index 1)

$$u = \frac{\langle\theta\rangle}{\theta_0}, \quad p = -\frac{du}{d\xi} \tag{1.9}$$

we reduce the problem of the propagation rate of a flame to the boundary problem

$$dp / du = \Phi / p - \omega, \; 0 < u < 1 \tag{1.10}$$

$$P(0) = 0 \tag{1.11}$$

$$P(1) = 0 \tag{1.12}$$

$$2\Phi = \begin{cases} (u + Fp)\exp\left[\frac{-\theta_0(u + Fp)}{1 - \sigma(u + Fp)}\right] + (u - Fp)\exp\left[\frac{-\theta_0(u - Fp)}{1 - \sigma(u - Fp)}\right], & 0 < u < \varepsilon \\ 0, & \varepsilon \leqslant u \leqslant 1 \end{cases} \tag{1.13}$$

Because Φ is even, the sign of the modulus p in Eq. (1.13) is not written. In the physical sense of the problem

$$\omega > 0, \; F > 0, \; \theta_0 > 0, \; \sigma = \beta\theta_0 = 1 - T_- / T_+ < 1$$

Since Eq. (1.10) is first order, in general, there does not exist a solution at once satisfying the two conditions (1.11) and (1.12) except, perhaps, for a few values of ω.

2. Existence and Uniqueness. Limits of Propagation

The point (0, 0) is a singular saddle point for Eq. (1.10). Two solutions emerge from it: $p_1(u)$ and $p_2(u)$. The slopes of the integral curves at the singular point are the roots of the equation

$$\lambda^2 + \omega\lambda - 1 = 0 \tag{2.1}$$

$$\frac{dp_1(0)}{du} = \lambda_1 = -\frac{\omega}{2} + \sqrt{\frac{\omega^2}{4} + 1}, \quad \frac{dp_2(0)}{du} = \lambda_2 = -\frac{\omega}{2} - \sqrt{\frac{\omega^2}{4} + 1}$$

Proposition 1. Suppose $(0, u_*)$ is the region of a noncontinuous solution ([9], p. 173) $p_1(u)$. Then, for ω and F, it can be shown that there is a $k = k(u_*, \omega, F) > 0$, such that in the interval $(0, u_*)$ the following inequality is satisfied:

$$p_1(u) - ku > 0 \tag{2.2}$$

We select k in such a way as to satisfy the inequality

$$L \equiv {}^1/_2(1 - Fk)\exp\left[\frac{-\theta_0(1 - Fk)u}{1 - \sigma(1 - Fk)u}\right] + {}^1/_2(1 + Fk)\exp\left[\frac{-\theta_0(1 + Fk)u}{1 - \sigma(1 + Fk)u}\right] - 2\omega k - k^2 > 0 \tag{2.3}$$

Since

$$L > {}^1/_2(1 - Fk)\exp\left[\frac{-\theta_0(1 - Fk)u}{1 - \sigma(1 - Fk)u}\right] - 2\omega k - k^2 > \{\text{for } (1 - Fk) > 0\} > (1 - Fk)A - 2\omega k - k^2, \quad 2A = \exp\left[\frac{-\theta_0 u_*}{1 - \sigma u_*}\right]$$

Eq. (2.3) holds if k satisfies the equation

$$(1 - Fk)A - 2\omega k - k^2 = 0 \tag{2.4}$$

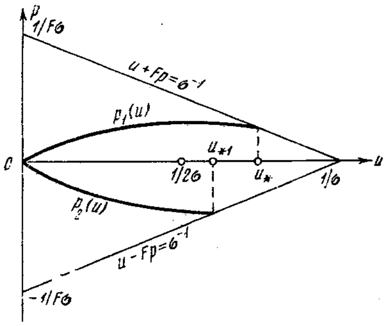

Fig. 1

where

$$k = -(\omega + AF/2) + \sqrt{(\omega + AF/2)^2 + A} \qquad (2.5)$$

is its positive root. We shall show that Eq. (2.2) is satisfied with this k. We rewrite Eq. (2.4) in the form

$$[k^2 + \omega k - 1] + [(\omega + AF)k + (1 - A)] = 0$$

We assume that $k \geq \lambda_1$, so that on the strength of (2.1), the expression in the first brackets is non-negative and in the second brackets is positive, which means that the left-hand part is strictly positive; but this is impossible, consequently, $k < \lambda_1$. Therefore, (2.2) is satisfied in the vicinity of $u = 0$. If it is assumed that for some $0 < u_0 < u_*$, the equality

$$p_1(u_0) - ku_0 = 0$$

holds, then, at the point u_0 it follows that

$$\frac{dp_1}{du} = \frac{\Phi(u_0, p_1(u_0))}{p_1(u_0)} - \omega \Big|_{p_1(u_0)=ku_0} \leqslant k$$

which is impossible, since the latter inequality violates (2.3).

Result. As $u \to u_*$ the point of the integral curve $(u, p_1(u))$ approaches without limit to the line $u + Fp = \sigma^{-1}$ (Fig. 1).

Actually, $p_1(u)$ is enclosed between the lines $p = 0$ and $u + Fp = \sigma^{-1}$, at which the right-hand part of (1.10) undergoes a discontinuity. As $u \to u_*$, the point of the integral curve $(u, p_1(u))$ should approach without limit to one of the given lines ([9], p. 175). Because of (2.2), this line cannot be $p = 0$.

Note. If $(0, u_{*1})$ is a region of the noncontinuous solution $p_2(u)$, the existence of a number $k_1 < 0$ (for example, the negative root of (2.4)) can be demonstrated which will satisfy the inequality $p_2(u) - k_1 u < 0$ in this region and at $u \to u_{*1}$ the point of the integral curve $(u, p_2(u))$ approaches without limit to the line $u - Fp = \sigma^{-1}$ (Fig. 1).

We shall study the behavior of the function Φ at the solution $p_1(u)$. Since $\Phi(0, p_1(0)) = 0$, and

$$\frac{d\Phi}{du} = \frac{\partial\Phi}{\partial u} + \frac{\partial\Phi}{\partial p_1}\frac{dp_1}{du}\Big|_{u=0} = 1$$

Φ is positive in the vicinity of $u = 0$. Let $\Phi > 0$ at $0 < u < u_0$ and $\Phi(u_0, p_{10}) = 0$, $p_{10} = p_1(u_0)$. Then, it is necessary that

$$\frac{d\Phi}{du}\Big|_{u=u_0} \leqslant 0 \qquad (2.6)$$

It is evident that

$$2\frac{\partial\Phi}{\partial u}(u_0, p_{10}) > \left\{1 - \frac{\theta_0(u_0 + Fp_{10})}{[1 - \sigma(u_0 + Fp_{10})]^2}\right\} \exp\left[\frac{-\theta_0(u_0 + Fp_{10})}{1 - \sigma(u_0 + Fp_{10})}\right] + 1$$

We rewrite the expression on the right-hand side of the inequality in the form

$$\varphi(\zeta) = 1 - (\beta\zeta^2 + \zeta - 1)\exp(-\zeta)$$
$$\zeta = \frac{\theta_0(u_0 + Fp_{10})}{1 - \sigma(u_0 + Fp_{10})}, \quad 0 < u_0 + Fp_{10} < \sigma^{-1}, \quad 0 < \zeta < \infty$$

The value $\varphi(\zeta)$ is non-negative, if

$$\inf_{\zeta > 0} \varphi = 1 - (4\beta + 1)e^{-2} \geqslant 0$$

or

$$\beta \leqslant {}^1/_4(e^2 - 1) \approx 1.597 \qquad (2.7)$$

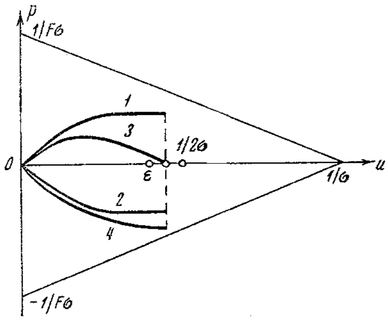

Fig. 2

It is also evident that

$$\frac{2}{F}\frac{\partial \Phi}{\partial p}(u_0, p_{10}) < \left\{1 - \frac{\theta_0 (u_0 + F p_{10})}{[1 - \sigma (u_0 + F p_{10})]^2}\right\} \exp\left[\frac{-\theta_0 (u_0 + F p_{10})}{1 - \sigma (u_0 + F p_{10})}\right] - 1$$

We represent the right-hand part of this inequality in the form

$$\psi(\zeta) = -1 - (\beta \zeta^2 + \zeta - 1) \exp(-\zeta) = \varphi(\zeta) - 2$$

It is not difficult to verify that $\psi(\zeta) < 0$ for $\zeta > 0$. Thus, if β satisfies the condition (2.7), then

$$\frac{\partial \Phi}{\partial u}(u_0, p_{10}) > 0, \quad \frac{\partial \Phi}{\partial p_1}(u_0, p_{10}) < 0$$

Considering that

$$\frac{dp_1}{du}(u_0) = -\omega$$

we obtain

$$\left.\frac{d\Phi}{du}\right|_{u=u_0} > 0$$

Since this inequality violates (2.6) this indicates that when satisfying the condition (2.7) $\Phi(u, p_1)$ cannot be reduced to zero. It is shown analogously that (2.7) is a sufficient condition for the positivity of Φ in the solution $p_2(u)$. The function Φ is not negative in a physical sense. Therefore, we shall henceforth, assume that β satisfies the condition (2.7). The physical restriction (2.7) is always satisfied since $\beta < 1$.

Proposition 2. If $(0, u_*)$ is the region of the noncontinuous solution $p_1(u)$, then $u_* \geq 0.5\ \sigma^{-1}$.

On the strength of the result of proposition 1

$$\lim_{u \to u_*} (u + F p_1) = \sigma^{-1}$$

or

$$\lim_{u \to u_*} F p_1 = \sigma^{-1} - u_*$$

Since

$$\lim_{u \to u_*} (u + F p_1) \exp\left[\frac{-\theta_0 (u + F p_1)}{1 - \sigma (u + F p_1)}\right] = 0$$

it follows that

$$\lim_{u \to u_*} \Phi = (u_* - 0.5\sigma^{-1}) \exp\left[\frac{-\theta_0 (u_* - 0.5\sigma^{-1})}{1 - \sigma u_*}\right]$$

If it is assumed that $u_* < 0.5\ \sigma^{-1}$, then, as seen from the latter expression, an area around the point $u = u_*$ is found in which Φ will be negative, but this is impossible because $\Phi(u, p_1) > 0$.

Note. If $(0, u_{*1})$ is the region of the noncontinuous solution $p_2(u)$ it can be shown that $u_{*1} \geq 0.5\ \sigma^{-1}$.

Result. For any $F > 0$, there exists a unique $\omega > 0$, for which a solution of Eq. (1.10) satisfies the conditions (1.11) and (1.12) if

$$0.5\ \sigma^{-1} \geqslant \varepsilon \tag{2.8}$$

The proof is conducted by the method of Ya. B. Zel'dovich [10]. We shall designate the solution of (1.10) with the condition (1.11) as $p_\varepsilon(u)$, where $0 < u < \varepsilon$. On the strength of Proposition 2 and (2.8), $u_* \geq \varepsilon$, so that the following limits make sense:

$$\lim_{u \to \varepsilon} p_{j\varepsilon}(u) = p_{j\varepsilon}(\varepsilon), \quad j = 1, 2$$

Since Φ is positive, $p_{1\varepsilon}(\varepsilon) > 0$ and $p_{2\varepsilon}(\varepsilon) < 0$ at $\omega = 0$. We shall introduce into the examination of the derivative of the solution p (now either $p_{1\varepsilon}$ or $p_{2\varepsilon}$)

$$q = \frac{\partial p(\omega, u)}{\partial \omega}$$

Since $p(\omega, 0) = 0$, then $q(0) = 0$. The equation for q is obtained by differentiating (1.10) with respect to ω

$$\frac{dq}{du} = q \frac{\partial}{\partial p}\left(\frac{\Phi}{p}\right) - 1$$

while its solution, taking into account the condition at u = 0, has the form

$$q = -\exp\left(\int_0^u \frac{\partial}{\partial p}\left(\frac{\Phi}{p}\right) du\right) \int_0^u \exp\left(-\int_0^u \frac{\partial}{\partial p}\left(\frac{\Phi}{p}\right) du\right) du$$

It follows from this that q is negative. The solutions of (1.10) with the condition (1.11) in the region (0, 1) are

$$p_j = \begin{cases} p_{j\varepsilon}(u), & 0 < u < \varepsilon \\ p_{j\varepsilon}(\varepsilon) - \omega(u - \varepsilon), & \varepsilon \leqslant u < 1 \end{cases}$$

where

$$p_j(\omega, 1) = p_{j\varepsilon}(\omega, \varepsilon) - \omega(1 - \varepsilon)$$

Since

$$\frac{dp_j(\omega, 1)}{d\omega} = q_j(\varepsilon) - (1 - \varepsilon) < -(1 - \varepsilon)$$

$p_j(\omega, 1)$ decreases monotonically, and without limit with an increase in ω. Since

$$p_1(0, 1) = p_{1\varepsilon}(0, \varepsilon) = -p_{2\varepsilon}(0, \varepsilon) = -p_2(0, 1) > 0$$

(Fig. 2, curves 1 and 2), $p_2(\omega, 1)$ remains negative with an increase in ω, while for some unique ω, $p_1(\omega, 1)$ is reduced to zero (Fig. 2, curves 3 and 4). Considering that $\varepsilon \approx 1$, we can represent the function (2.8) in the form of an approximate inequality $\sigma \leq 0.5$.

We shall now examine the case when

$$0.5\ \sigma^{-1} < \varepsilon \tag{2.9}$$

Suppose $(0, u_{*1})$ is the region of the noncontinuous solution $p_2(u)$. Since as $u \to u_{*1}$ the point $(u, p_2(u))$ approaches without limit to the line $u - Fp = \sigma^{-1}$, then then

$$u_{*1} - Fp_2(\omega, u_{*1}) = \sigma^{-1}, \quad p_2(\omega, u_{*1}) = \lim_{u \to u_{*1}} p_2(\omega, u) \tag{2.10}$$

Considering u_{*1} as a function of ω, after differentiating (2.10) with respect to ω, we find

$$\frac{du_{*1}}{d\omega} = Fq_2(u_{*1})\left[1 - F\frac{dp_2}{du}(u_{*1})\right]^{-1} < 0 \tag{2.11}$$

since $q_2(u_{*1}) < 0$ and $\dot{p}_2(u_{*1}) < 0$. The inequality (2.11) means that u_{*1} falls off monotonically with an increase in ω. If $u_{*1} < \varepsilon$ at $\omega = 0$, then, on the basis of (2.11) the inequality is also retained for $\omega > 0$. In this case, the solution $p_2(u)$ does not exist in the region (0, 1). If $u_{*1} > \varepsilon$ at $\omega = 0$, then, as already shown, $p_2(\omega, 1)$ decreases monotonically with increasing ω and remaining negative as long as $u_{*1} < \varepsilon$ does not occur. Thus, it is proven that for any ω, either $p_2(u)$ does not exist at (0, 1) or $p_2(\omega, 1)$ is negative. Therefore, we shall restrict the further examination to the solution $p_1(u)$.

Proposition 3. For any F, an ω can be found such that a solution $p_1(u)$ will exist in the region (0, 1).

For proof it is necessary to demonstrate the existence of an ω, such that $u_* \geq \varepsilon$. It is evident that

$$\max_{0<u<\varepsilon} \Phi \leqslant \max_{y>0}\left[y \exp\left(\frac{-\theta_0 y}{1 - \sigma y}\right)\right] = \frac{2}{\theta_0(1 + 2\beta + \sqrt{1+4\beta})} \exp\left(\frac{-2}{1 + \sqrt{1+4\beta}}\right) = \Phi_m$$

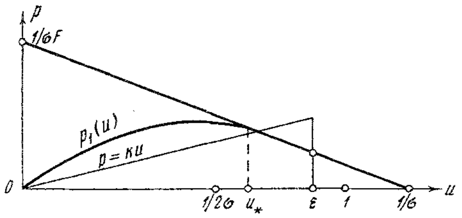

Fig. 3

The solution of the equation

$$\frac{dp_3}{du} = \frac{\Phi_m}{p_3} - \omega, \quad p_3(0) = 0 \tag{2.12}$$

has the upper limit $p_1(u)$ ([11], p. 268). It is not difficult to show that $p_3 < \Phi_m/\omega$, which means that $P_3 < \Phi_m/\omega$. We select ω in such a way that $p = \Phi_m/\omega$ and $u = \varepsilon$ intersect at a point not higher than the point of intersection of the lines $u = \varepsilon$ and $u + Fp = \sigma^{-1}$. For this, it is necessary that

$$\Phi_m/\omega \leqslant F^{-1}(\sigma^{-1} - \varepsilon)$$

from which

$$\omega \geqslant F\Phi_m \sigma (1 - \sigma\varepsilon)^{-1}$$

For such an ω in the region $(0, \varepsilon)$ the point $(u, p_1(u))$ can approach without limit to the line $u + Fp = \sigma^{-1}$ with the condition that the integral curve $p_1(u)$ and the line $p = \Phi_m/\omega$ intersect. But this is impossible, which means that $u_* \geq \varepsilon$.

Note. On the basis of Proposition 3 a solution $p_1(u)$ can always be constructed in the region $(0, 1)$. If $p_1(\omega, 1) > 0$, then, condition (1.12) can always be satisfied by increasing ω. If $p_1(\omega, 1) < 0$, it is natural to decrease ω. However, the case is possible, where $p_{1\varepsilon}(u)$ reaches the line $u + Fp = \sigma^{-1}$ before $p_1(\omega, 1)$ is reduced to zero. In other words, the case is possible, where ω cannot be taken so that either $u_* < \varepsilon$, or $p_1(\omega, 1) < 0$, i.e., an ω does not exist for which $p_1(\omega, 1) = 0$. Since the inequality (2.2) is of interest only within the limits of the region $(0, \varepsilon)$, from now on, we shall assume that in (2.5)

$$2A = \exp\left(\frac{-\theta_0\varepsilon}{1 - \sigma\varepsilon}\right)$$

We shall demonstrate the existence of an F_1, such that for any $F \leq F_1$ there is a unique ω, for which the solution of Eq. (1.10) satisfies the conditions (1.11) and (1.12). Let us examine the system

$$\frac{\Phi_m}{\omega} = F^{-1}(\sigma^{-1} - \varepsilon) \tag{2.13}$$

$$k\varepsilon - \omega(1 - \varepsilon) \geqslant 0 \tag{2.14}$$

and its solution

$$\omega = F\Phi_m \sigma (1 - \sigma\varepsilon)^{-1}$$

$$F \leqslant (1 - \sigma\varepsilon)\varepsilon A^{1/2} \{\Phi_m \sigma (1 - \varepsilon) [\Phi_m \sigma (1 + \varepsilon) + A\varepsilon (1 - \sigma\varepsilon)]\}^{-1/2} = F_1$$

In realizing (2.13), the function $p_1(u)$ exists in all of the region $0 < u < 1$. In light of (2.2), it follows from (2.14) that

$$p_{1\varepsilon}(\omega, \varepsilon) - \omega(1 - \varepsilon) > 0$$

or

$$p_1(\omega, 1) > 0$$

By increasing ω, one can satisfy (1.12) at some unique ω.

We now demonstrate the existence of an F_2, such that if $F \geq F_2$, a solution of Eq. (1.10) satisfying the conditions (1.11) and (1.12) does not exist for any ω. Let us examine the system

$$F\omega(1 - \varepsilon) = (\sigma^{-1} - \varepsilon) \tag{2.15}$$

$$Fk\varepsilon - (\sigma^{-1} - \varepsilon) \geqslant 0 \tag{2.16}$$

which for condition (2.9) has the solution

$$\omega = (1 - \sigma\varepsilon)[F\sigma(1 - \varepsilon)]^{-1}$$

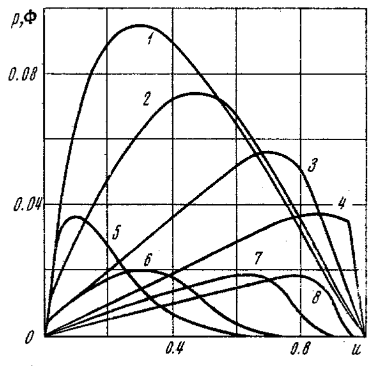

Fig. 4

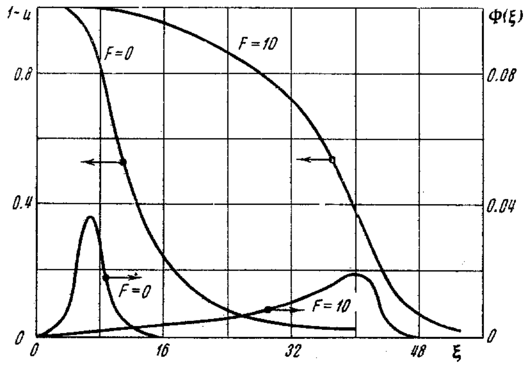

Fig. 5

$$F \geqslant \frac{1-\sigma\varepsilon}{\sigma}\left[\frac{1+\varepsilon}{A\varepsilon(1-\varepsilon)(2\varepsilon-\sigma^{-1})}\right]^{1/2} = F_2$$

Realization of the inequality (2.16) shows that the straight lines $p = ku$ and $u = \varepsilon$ intersect at a point not lying below the line $u + Fp = \sigma^{-1}$ (Fig. 3). This means that $u_* < \varepsilon$. With an increase in ω, one could reach a point where $u_* \geq \varepsilon$ (see Proposition 3), i.e., a position such that $p_{1\varepsilon}(\varepsilon) \leq F^{-1}(\sigma^{-1} - \varepsilon)$, but in this case, $\omega(1-\varepsilon) > F^{-1}(\sigma^{-1} - \varepsilon)$ and $p_{1\varepsilon}(\varepsilon) - \omega(1-\varepsilon) < 0$. The situation arises which was spoken of in the note to Proposition 3. Since the value of ε is close to unity, we shall substitute the inequality $\sigma > 0.5$.

Suppose the set $\{F_1\}$ is such that, if $F \in \{F_1\}$, then, for it there exists a unique ω for which the solution (1.10) satisfies the conditions (1.11) and (1.12).* In view of the existence of F_2 the set $\{F_1\}$ is bounded from above, and consequently, has a precise upper bound. We shall call $F_* = \sup\{F_1\}$ the critical value; it determines the limit of propagation of a turbulent flame.

Therefore, as a result of the analysis, we obtain the following:

1) Φ is nonnegative, if $\beta \leq 0.25\,(e^2 - 1) \approx 1.597$;

2) the problem of determining the rate of burning has a unique solution for any F, when $0 \leq \sigma \leq 0.5$; for $0.5 < \sigma < 1$ the solution exists only for $0 \leq F < F_*$, where $F_* = \sup\{F_1\}$.

3. Analysis of Results

Computations were made of the problem (1.10)-(1.12) on a computer in order to calculate the quantitative characteristics of turbulent burning and the structure of the flame front. The range of variation of the parameters is

$$6 \leqslant \theta_0 \leqslant 14,\ 0 \leqslant \sigma \leqslant 0.9,\ 0 \leqslant \beta \leqslant 0.15$$

For $0 \leq \sigma \leq 0.5$, $0 \leq F < \infty$, while the limiting values of F_* were sought for $\sigma > 0.5$.

Integral curves of p(u) and functions of the variation in heat release $\Phi(u)$ at $\theta_0 = 10$, $\beta = 0$, and $\varepsilon = 0.95$ for different values of F are presented in Fig. 4. Curves 1, 2, 3, and 4 correspond to p(u) with different values of F: F = 3, 5, 10, and 20, while curves 5, 6, 7, and 8 correspond to $\Phi(u)$ with the same values of F. In contrast to a laminar flame (F = 0), a turbulent flame ($F \neq 0$) is expanded (the maximum gradient p_m decreases with a growth in F), while the maximum averaged heat release is significantly shifted in the direction of the initial temperature.

The reaction proceeds near the initial temperature, and the preheating zone in a turbulent flame is far narrower than the reaction zone. This is seen from Fig. 5, in which the distributions of temperature u and heat release Φ are given as functions of ξ (for F = 0 and F = 10, $\theta_0 = 10$, $\beta = 0$). (The origin of the reading in Fig. 5 is conventional, since Eq. (1.1) is invariant relative to the transformation $\xi \pm$ const.)

Asymptotic behavior of Φ_m is observed for F > 4. The variation functions of the characteristic values of burning: p_m, Φ_m, and their locations u_p and u_Φ are shown in Fig. 6 along with the propagation rate ω_1 as a function of F. Analogous qualitative results were obtained also for $\sigma \neq 0$.

*It can be shown that $\{F_1\}$ as a set on the numerical axis is connected.

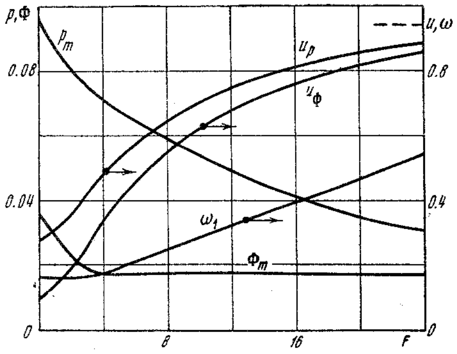

Fig. 6

TABLE 1

σ	θ_1	F_*	τ_1/τ_+	τ_1/τ_1	ω_1	ω_*
0.7	6	10.5	110	5.17	0.2168	0.4490
	14	14	196	1.92	0.0984	0.2336
0.9	6	5.25	27.6	1.11	0.2004	0.2772
	14	8.25	68.1	0.61	0.0948	0.1581

We shall make an estimate of the propagation rate of a flame ω_1 for $F \gg 1$ and $\theta_0 \gg 1$. In this case, in the vicinity of u, close to the cut-off parameter ε, $u + Fp \approx \sigma^{-1}$, so that Φ can be determined by the approximative equality

$$2\Phi = (u - Fp)\exp\left[\frac{-\theta_0(u - Fp)}{1 - \sigma(u - Fp)}\right], \quad p > 0 \tag{3.1}$$

Hence, it follows that Φ_m is attained at

$$(u - Fp)_m = 2\theta_0^{-1}(1 + 2\beta + \sqrt{1 + 4\beta})^{-1} \tag{3.2}$$

and is equal to half the Φ_m for laminar burning

$$\Phi_m = \theta_0^{-1}(1 + 2\beta + \sqrt{1 + 4\beta})^{-1}\exp\left(\frac{-2}{1 + \sqrt{1 + 4\beta}}\right) \tag{3.3}$$

Since for $F \gg 1$ the maxima p_m and Φ_m approach each other asymptotically (Fig. 6), while $u \to \varepsilon$, it follows from (1.10) taking (3.2) and (3.3) into account that:

$$\omega_1(F \gg 1) = F[\theta_0\varepsilon(1 + 2\beta + \sqrt{1 + 4\beta}) - 2]^{-1}\exp\left(\frac{-2}{1 + \sqrt{1 + 4\beta}}\right) \tag{3.4}$$

For example, for $\theta_0 = 10$, $\beta = 0$, $\varepsilon = 0.95$, and $F = 26$ for (3.4), we obtain $\omega_1 = 0.564$, while a calculation on the computer gives $\omega_1 = 0.590$ (about 5% disagreement).

On the basis of an analysis of the multiple variants of the calculation, it was established that for $F \geq 5$, ω_1 is represented by the linear dependence $\omega_1 = N + MF$, with an error no worse than 1%, where N and M are functions of θ_0 and σ.

In physical variables the kinetic propagation rate of a flame is written in the form

$$w_1 = \frac{N}{\omega_0}\sqrt{1 + \frac{l_1\sqrt{\overline{w'^2}}}{a_0}}\, w_0 + \frac{M}{\omega_0^2}\frac{l_1 w_0^2}{a_0} \tag{3.5}$$

For $F > 5$, $\tau_1/\tau_+ > 25$, therefore, keeping in mind the relay mechanism of the transmission of burning [7], we obtain

$$w_\Sigma = \sqrt{\overline{w'^2}} + \frac{N}{\omega_0}\sqrt{1 + \frac{l_1\sqrt{\overline{w'^2}}}{a_0}}\, w_0 + \frac{M}{\omega_0^2}\frac{l_1 w_0^2}{a_0} \tag{3.6}$$

Here, w_Σ is the total propagation rate of the flame, and N/ω_0 and M/ω_0^2 are functions of θ_0 and σ. The interpolation equation (3.6) linearly "unifies" the earlier known mechanisms of burning: the first term gives the contribution of the surface model [7], the second gives that of the volumetric model [5], and the third the microvolumetric [5] or focus model [12]. For $F \gg 1$ and $\sigma \leq 0.5$ burning takes place mainly by the microvolumetric mechanism.

The limiting values of F_* and ω_* as well as the parameters τ_1/τ_+ and τ_1/τ_0 are found for $0.7 \leq \sigma \leq 0.9$ (Table 1). Their values depend on the kinetics of the chemical reaction.

In conclusion, we note that the stability of the turbulent burning model under examination was not studied in this work. It is possible that in the region $0 \leq \sigma \leq 0.5$ the requirement of stability sets a limit on the parameter F.

4. Algorithm for Calculating ω

If ω satisfies the equation

$$\Phi_m / \omega - \omega (1 - \varepsilon) = 0 \tag{4.1}$$

then, from the inequality $p_{1\varepsilon} - \Phi_m/\omega < 0$, we have

$$p_1(\omega, 1) = p_{1\varepsilon}(\omega, \varepsilon) - \omega(1 - \varepsilon) < 0$$

and, consequently, the solution of (4.1) gives the upper boundary of the interval for the root of the equation

$$p_1(\omega, 1) = 0 \tag{4.2}$$

The desired ω is included within the limits $0 < \omega < \sqrt{\Phi_m/(1-\varepsilon)}$. The root of Eq. (4.2) is found by the method of equal partition. Suppose that in the s-th step the interval (ω_A^s, ω_B^s) containing the root of (4.2) is obtained. Then

$$(\omega_A^{s+1}, \omega_B^{s+1}) = \begin{cases} (\omega_A{}^s, \alpha), & \text{if} \quad p_1(\alpha, 1) < 0 \\ (\alpha, \omega_B{}^s), & \text{if} \quad u_* < \varepsilon \quad \text{or} \quad p_1(\alpha, 1) > 0 \end{cases}$$
$$\alpha = 0.5(\omega_A{}^s + \omega_B{}^s)$$

Equation (1.10) is solved by the method of differences. The calculation leads to the realization of the inequality

$$\omega_B{}^s - \omega_A{}^s < \delta \tag{4.3}$$

In all the calculations it was assumed that $\delta = 10^{-4}$. If the inequality (4.3) is satisfied and $p_1(\omega_A^s, 1) > 0$, $p_1(\omega_B^s, 1) < 0$, then, it is assumed that $F < F_*$, while if (4.3) is satisfied, and $p_{1\varepsilon}(\omega_A^s, u)$ extending up to $u_* < \varepsilon$, then, $F > F_*$.

LITERATURE CITED

1. Ya. B. Zel'dovich, "Theory of the burning of unmixed gases," Zh. Tekhn. Fiz., 19, No. 10 (1949).
2. Th. von Karman and G. Millan, "Thermal theory of a laminar flame front near a cold wall," Fourth Sympos. (Internat.) on Combust., Cambridge, 1952, Williams and Wilkins Co. (1953).
3. L. A. Vulis, "Effect of temperature pulsations on the rate of turbulent burning," Izv. Akad. Nauk KazSSR, Ser. Énerg., No. 1 (1959).
4. K. I. Shchelkin, "The hydrodynamics of burning," Fizika Goreniya i Vzryva, 4, No. 4 (1968).
5. E. S. Shchetinkov, Physics of the Burning of Gases [in Russian], Nauka, Moscow (1965).
6. V. N. Vilyunov and A. A. Dvoryashin, "An Experimental Investigation of the Erosive Burning Effect," Fizika Goreniya i Vzryva, 7, No. 1 (1971).
7. K. I. Shchelkin and Ya. K. Troshin, Gasdynamics of Combustion [in Russian], Izd. AN SSSR, Moscow (1963).
8. L. S. G. Kovasznay, "A comment on turbulent combustion," Jet Propulsion, 26, No. 6 (1956).
9. L. S. Pontryagin, Simultaneous Differential Equations [in Russian], Nauka, Moscow (1965).
10. Ya. B. Zel'dovich, "Theory of propagation of a flame," Zh. Fiz. Khim., 22, No. 1 (1948).
11. I. S. Berezin and N. P. Zhidkov, Methods of Calculation [in Russian], Vol. 2, Fizmatgiz, Moscow (1962).
12. A. S. Sokolik, V. P. Karpov, and E. S. Semenov, "Turbulent burning of gases," Fizika Goreniya i Vzryva, 3, No. 1 (1967).